The Development of a Quantum Computer in Silicon

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December 4th, 2013
Outline

- Short history of development of quantum computing
- Comparison of classical and quantum computing
- Experimental requirements for the practical realisation of a quantum computer: the 7 DiVincenzo criteria
- Leading contenders in the solid state
- Current status of silicon based quantum computing
1982: Richard Feynman, at the First Conference on the Physics of Computation held at MIT, proposed a basic model for a quantum computer that would exploit the potential of massive quantum parallelism.

1994: Peter Shor discovers the factorisation algorithm for large numbers theoretically capable of breaking today's public key cryptosystems.

1995: Peter Shor and Andrew Steane simultaneously proposed the first schemes for quantum error correction.

1996 Lou Grover propose an exhaustive search algorithm that showed for a system of $n$ possibilities you can find the answer in $\sqrt{n}$ look-ups quantum mechanically compared with $n/2$ classically.

1998 Ray LaFlamme experimentally demonstrates error correction in a trichloroethylene molecule using liquid state NMR.
## Classical versus quantum bits

<table>
<thead>
<tr>
<th>Conventional Computer</th>
<th>Quantum Computer</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, 1</td>
<td></td>
</tr>
</tbody>
</table>

- **bits**
  - Information is stored in “bits”.
  - A bit can be either 0 or 1.

- **qubits**
  - Information is stored in “quantum bits” or qubits, which can be a combination of 0 and 1.

Quantum state of a two-level system such as spin or charge of $^{31}\text{P}$ nucleus:

- Charge:
  - $\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$

- Spin:
  - $\pi/2$ pulse:
    - $\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$
Classical versus quantum computation

Classical computer - can check many different possibilities in *rapid succession*
Quantum computer - can check many different possibilities *in parallel*

Digital information : 0
Superposition, 1 spin:
\[ \Psi = \alpha_1 |0\rangle + \alpha_2 |1\rangle \]
Entanglement, 2 spins:
\[ \Psi = \alpha_1 |00\rangle + \alpha_2 |01\rangle + \alpha_3 |10\rangle + \alpha_4 |11\rangle \]

<table>
<thead>
<tr>
<th># qubits</th>
<th>classical possibilities</th>
<th>power</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 or 1</td>
<td>2</td>
</tr>
<tr>
<td>1 1</td>
<td>00, 01, 10, 11</td>
<td>4</td>
</tr>
<tr>
<td>1 1 1</td>
<td>000, 001, 010, 011</td>
<td>8</td>
</tr>
<tr>
<td>1 1 1</td>
<td>100, 101, 110, 111</td>
<td></td>
</tr>
</tbody>
</table>

Quantum computer’s power *doubles* every time another qubit is added

A 30-qubit quantum computer would be more powerful than a supercomputer.

As for 300 qubits....
Difficult problems: the travelling salesman

Problem: A salesman has to travel to many cities and wants to work out the shortest possible route

14 cities: $10^{11}$ routes
for a classical 1GHz computer (10⁹ operations/sec) it would take 100 seconds

22 cities: $10^{19}$ routes
it would take 1600 years

28 cities
What can quantum computers do?

Quantum computers will not necessarily outperform classical computers but need to use algorithms that exploit quantum parallelism.

Applications: physical modelling (climate, engineering); simulations (chemistry, materials), database searching (bioinformatics); factorisation (data security)

Algebraic and Number Theoretic Algorithms (11 algorithms); e.g. factorising

Oracular Algorithms (29 algorithms); e.g. searching, linear differential equations

Approximation and Simulation Algorithms (10 algorithms); e.g. simulation, adiabatic algorithms
Experimental Requirements for Quantum Computing Devices
Relaxation and coherence times

The longitudinal relaxation time, $T_1$, is the time-scale for the exponential decay of a non-equilibrium polarization of spins to give up its Zeeman energy to the lattice.

→ it represents the maximum time available for a quantum computation

The transverse relaxation time, $T_2$

The amplitude of the net transverse magnetisation decays as the magnetic moments move out of phase with one another (shown by the small black arrows). Arise from spin-spin interactions.

The overall term for the observed loss of phase coherence is $T_{2^*}$ relaxation, which combines the effect of $T_2$ relaxation and additional de-phasing caused by local variations (inhomogeneities) in the applied magnetic field, e.g. by the presence of other nuclear spins.
Overview: Qubits in the Solid State

<table>
<thead>
<tr>
<th></th>
<th>Atom, molecule, ion</th>
<th>Electron spin</th>
<th>Nuclear spin</th>
<th>Superconducting qubit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>~10^{-10} m</td>
<td>~10^{-10} m (impurities)</td>
<td>&lt;10^{-10} m</td>
<td>~10^{-6} m</td>
</tr>
<tr>
<td>Energy gap</td>
<td>10^5–10^6 GHz, ~GHz (Rydberg atoms)</td>
<td>1–10 GHz</td>
<td>1–10 MHz</td>
<td>1–20 GHz</td>
</tr>
<tr>
<td>Frequency range</td>
<td>Optical, microwave</td>
<td>Microwave</td>
<td>Microwave</td>
<td>Microwave</td>
</tr>
<tr>
<td>Operating temperature</td>
<td>nK to μK</td>
<td>~100 mK (quantum dot), room temperature (NV center)</td>
<td>~mK</td>
<td>~10 mK</td>
</tr>
<tr>
<td>Single-qubit gate operation time τ₁</td>
<td>~μs (atom)</td>
<td>~10 ns</td>
<td>&gt;10 μs</td>
<td>~1 ns</td>
</tr>
<tr>
<td>Two-qubit gate operation time τ₂</td>
<td>~μs (atom)</td>
<td>~0.2 ns</td>
<td>~10 ms</td>
<td>~10–50 ns</td>
</tr>
<tr>
<td>Coherence time T₂</td>
<td>ms to s</td>
<td>ms to s</td>
<td>~s</td>
<td>~10–100 μs</td>
</tr>
<tr>
<td>T₂/τ₁</td>
<td>10–10⁴</td>
<td>10⁵–10⁸</td>
<td>10⁶</td>
<td>10⁴–10⁵</td>
</tr>
<tr>
<td>Coupling type</td>
<td>Electric or magnetic</td>
<td>Magnetic or electric</td>
<td>Magnetic</td>
<td>Electric or magnetic</td>
</tr>
<tr>
<td>Coupling strength with the cavity</td>
<td>&lt;kHz (B field), ~10 kHz (E field),</td>
<td>&gt;MHz (quantum dot)</td>
<td>~0.1 Hz</td>
<td>~0.1–1 GHz</td>
</tr>
</tbody>
</table>

- Superconducting qubits offer flexibility and strong coupling to external fields BUT have relatively short coherence times (<0.1ms)
- Microscopic systems are given by nature and can easily be made identical with long coherence times (>1ms) BUT they operate slowly due to weak coupling to external fields.

DiVincenzo Criteria for a scalable system

A quantum register of multiple qubits must be prepared in an addressable form and isolated from environmental influences, which cause the delicate quantum states to decohere.

Although weakly coupled to the outside world, the qubits must nevertheless be strongly coupled together to perform logic-gate operations.

There must be a readout method to determine the state of each qubit at the end of the computation.

1. A scalable physical system of well-characterized qubits

Well defined two level quantum system

| Physical system | Name                        | Information support          | |0>          | |1>          |
|-----------------|-----------------------------|------------------------------|-------------|-------------|
| Photon          | Polarisation encoding       | Polarisation of light        | Horizontal  | Vertical    |
|                 | Number of photons           | Fock state                   | Vacuum      | Single photon of light |
|                 | Time-bin encoding           | Time of arrival              | Early       | Late        |

Beyond CMOS: Emerging Materials and Devices
1. A scalable physical system of well-characterized qubits

2. The ability to initialize the state of the qubits to a simple state

\(^1\) D. DiVincenzo, Fortschritte der Physik-Progress of Physics 48, 771 (2000)
Initialisation of electron spin

$|0\rangle$ $|1\rangle$

$\text{SET}$ $\text{P Donor}$

$\text{SET-island}$

$\text{P}$ $\text{SET}$ $\text{drain}$

$\text{B}$
DiVincenzo Criteria for a scalable system

1. A scalable physical system of well-characterized qubits

2. The ability to initialize the state of the qubits to a simple state

3. Coherence times $>>$ gate-operation times

$^{1}$ D. DiVincenzo, Fortschritte der Physik-Progress of Physics 48, 771 (2000)
### Table 1 | Current performance of various qubits

<table>
<thead>
<tr>
<th>Type of qubit</th>
<th>$T_2$</th>
<th>Benchmarking (%):</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>One qubit</td>
<td>Two qubits</td>
</tr>
<tr>
<td>Infrared photon</td>
<td>0.1 ms</td>
<td>0.016</td>
<td>1</td>
</tr>
<tr>
<td>Trapped ion</td>
<td>15 s</td>
<td>0.48†</td>
<td>0.7*</td>
</tr>
<tr>
<td>Trapped neutral atom</td>
<td>3 s</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Liquid molecule nuclear spins</td>
<td>2 s</td>
<td>0.01†</td>
<td>0.47†</td>
</tr>
<tr>
<td>$e^-$ spin in GaAs quantum dot</td>
<td>3 μs</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$e^-$ spins bound to $^{31}$P:$^{28}$Si</td>
<td>0.6 s</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$^{29}$Si nuclear spins in $^{28}$Si</td>
<td>25 s</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>NV centre in diamond</td>
<td>2 ms</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Superconducting circuit</td>
<td>4 μs</td>
<td>0.7†</td>
<td>10*</td>
</tr>
</tbody>
</table>

Measured $T_2$ times are shown, except for photons where $T_2$ is replaced by twice the hold-time (comparable to $T_1$) of a telecommunication-wavelength photon in fibre. Benchmarking values show approximate error rates for single or multi-qubit gates. Values marked with asterisks are found by quantum process or state tomography, and give the departure of the fidelity from 100%. Values marked with daggers are found with randomized benchmarking. Other values are rough experimental gate error estimates. In the case of photons, two-qubit gates fail frequently but success is heralded; error rates shown are conditional on a heralded success. NV, nitrogen vacancy.

DiVincenzo Criteria for a scalable system

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2. The ability to initialize the state of the qubits to a simple state

3. Coherence times $>>$ gate-operation times

4. A universal set of quantum gates

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Universal quantum gates

Universal: one single computer for different computational tasks

In quantum computation all operations must be reversible.

An example of a non-reversible gate is an AND gate where two inputs only give one output → therefore information is lost.

The quantum states of a qubit are a vector in 2D complex vector space.

\[ \Psi = \alpha |0\rangle + \beta |1\rangle \]

A superposition is a linear combination of the 0 and 1 state amplitude with coefficients \( \alpha \) and \( \beta \). The constraints are that:

\[ |\alpha|^2 + |\beta|^2 = 1 \]
Single quantum NOT gate

Quantum NOT gate: $|0\rangle \rightarrow |1\rangle$ and $|1\rangle \rightarrow |0\rangle$

But we also have a superposition so

$\alpha|0\rangle + \beta |1\rangle \rightarrow \alpha|1\rangle + \beta |0\rangle$

Logic gate

\[
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\]

\[
X = \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\]

$X |0\rangle = \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix} \begin{pmatrix}
1 \\
0
\end{pmatrix} = \begin{pmatrix}
0 \\
1
\end{pmatrix} = |1\rangle$

$X |1\rangle = \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix} \begin{pmatrix}
0 \\
1
\end{pmatrix} = \begin{pmatrix}
1 \\
0
\end{pmatrix} = |0\rangle$
2 qubit controlled NOT gate (CNOT)

4 computational basis states:
\[ \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle \quad \alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1 \]

If the control is 1, flip the target qubit; otherwise do nothing.

- \[ |00\rangle \rightarrow |00\rangle \]
- \[ |01\rangle \rightarrow |01\rangle \]
- \[ |10\rangle \rightarrow |11\rangle \]
- \[ |11\rangle \rightarrow |10\rangle \]
A universal set of gate operations

A-gates control the interaction between a nuclear spin qubit and the electron spin.

\[ B_{ac} \]

J-gates control the exchange interaction between electron spins.

\[ \Rightarrow \text{effectively using an electron spin mediated nuclear spin – nuclear spin interaction.} \]
DiVincenzo Criteria for a scalable system

1. A scalable physical system of well-characterized qubits

2. The ability to initialize the state of the qubits to a simple state

3. Coherence times >> gate-operation times

4. A universal set of quantum gates

5. A qubit-specific measurement capability

\[^{1}\text{D. DiVincenzo, Fortschritte der Physik-Progress of Physics 48, 771 (2000)}\]
Single shot spin read out

DiVincenzo Criteria for a scalable system

1. A scalable physical system of well-characterized qubits
2. The ability to initialize the state of the qubits to a simple state
3. Coherence times >> gate-operation times
4. A universal set of quantum gates
5. A qubit-specific measurement capability
6. The ability to interconvert stationary and flying qubits

\(^1\) D. DiVincenzo, Fortschritte der Physik-Progress of Physics 48, 771 (2000)
The ability to interconvert stationary and flying qubits

Optically addressing dopant atoms in silicon


Semiconductor nanophotonics

R. Van Meter et al., Int. J QC 1, 295 (2010).
DiVincenzo Criteria for scalable system

1. A scalable physical system of well-characterized qubits

2. The ability to initialize the state of the qubits to a simple state

3. Coherence times $>>$ gate-operation times

4. A universal set of quantum gates

5. A qubit-specific measurement capability

6. The ability to interconvert stationary and flying qubits

7. The ability to faithfully transmit flying qubits between specified locations

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Faithfully transmit flying qubits

Hybrid proposals


Spin hybrid quantum circuits with spin and superconducting qubits
Leading Contenders in the Solid State
Electron spins in GaAs

Demonstration of two qubit gate in singlet-triplet basis
$T_2 \approx 200\mu s$

Bell state fidelity $\approx 0.72$


Demonstration of flying qubits:
Transport and manipulation of qubits over 6 microns in 40ps using Aharonov-Bohm rings connected to channel wires


Main limitation is coherence times $\approx$ hundreds of microseconds or less
Diamond based qubits

Two qubit parity measurement on nuclear spins in NV centres exploiting electron spin as a read-out ancilla


Demonstration of room temperature entanglement of 2 NV centres

Entanglement fidelity \( \approx 0.67 \)

F. Dolde et al., Nature Physics 9, 139 (2013).

Scalable architectures:
L. Childress et al., PRL 96, 070504 (2006)
P. Rabl et al., Nat Phys 6, 602 (2010)

Main limitation is difficulty of reproducible fabrication

\( T_2 \approx 400 \mu s \) (PRB 2011)
Superconducting qubits

$T_2 \sim 100 \text{ µs}$

Single qubit gate time $\sim 1$ns

Two qubit gate times $\sim 10$-50ns
Current Status of Silicon Quantum Computing
Silicon based qubits

**P nuclear spin qubit**

$^{nat}Si \ T_2 (n) > 60 \text{ ms}$ (ionised donor)

Nuclear spin read-out fidelity 99.8%

*J. Pla et al., Nature 496, 334 (2013)*

**Electron spin qubit in Si/SiGe**

Singlet-triplet basis

$^{nat}Si \ T_2 (e) > 360\text{ns}$

*B.M. Maune et al., Nature 481, 344 (2012)*

Main limitation is difficulty of fabrication at such small scales
First proposal for a silicon quantum computer

Qubits are the nuclear spins of $^{31}\text{P}$ donor atoms in $^{28}\text{Si}$

**Advantages:**
- relaxation $T_1$ long ($10^{18}$ s)
- Low spin-orbit coupling
- Spin free host with low abundance of $^{29}\text{Si}$ (~5%)
- compatible with existing multi-billion dollar silicon microelectronics industry and scaleable

**Disadvantages:**
- require the ability to dope Si with atomic precision aligned to nanometer sized surface gates

Spin Coherence of P donors

$T_2$ increases as $n_s$ is reduced

Bulk measurements:
$^{28}\text{Si} T_1(e) \sim 1 \text{ hour (1.2K; 0.35T)}$
$^{28}\text{Si} T_2(e) \sim \text{ secs (}^{28}\text{Si, 1.2K)}$


$^{31}\text{P}$ nuclear memory: $T_2(n) \sim 180 \text{ seconds}$

$^{28}\text{Si}: \text{“Semiconductor Vacuum”}$

M. Steger et al., Science 336, 1280 (2012)
J.L. Morton et al., Nature 455, 7216 (2008)

Ionised donor $\sim 39 \text{ mins (RT)}$

M. Saeedi et al., Science 342, 130 (2013)
Scalable 2D architecture

Shuttling time ~ns

Donor based qubits by ion implantation

Andrew Dzurak, Andrea Morello and David Jamieson
Atomic Fabrication Strategy in Silicon

1. Fabrication of registration markers
2. PH$_3$ Dosing
3. Si Growth
4. Barrier growth
5. Hydrogen Desorption
6. P Incorporation
7. Buried dopant imaging
8. Registered surface gates
Narrowest, lowest resistance conducting Si nanowires

- Lowest resistivity doped silicon wires
- Constant resistivity down to ~1.7nm
- Resistivity comparable to bulk doping of similar density, $\rho \sim 0.3 \times 10^{-3} \ \Omega \text{cm} $ (4.2K)


First deterministic, precision single donor device


Ejected Si at the same site after incorporation
First deterministic, precision single donor device

$E_C = 47\pm 2\text{meV}$

$1_s(T_1) = 11.4\pm 1$

$1_s(E) = 15\pm 2$

Compared well with:

$1_s(T_1) = 11.7$

$1_s(E) = 13.1$

Single shot spin read-out using all epitaxial SETs
Integration of an in-plane detector for spin read-out

Single shot spin read-out: spin-up

Single shot spin read-out: spin down

Spin relaxation rates, $T_1$

$T_1^{-1}(B) \approx B^5$ agrees with spin-lattice relaxation mechanism from valley depopulation

P donor single atom qubit

$^{\text{nat}}\text{Si } T_2 (\text{e}) > 200 \ \mu\text{s}$
(Hahn echo)

Electron spin read-out fidelity \( \sim 57\% \)


$^{\text{nat}}\text{Si } T_2 (\text{n}) > 60 \ \text{ms}$
(ionised donor)

Nuclear spin read-out fidelity \( \sim 99.8\% \)

J. Pla et al., Nature 489, 541 (2012)

Summary

• Quantum computing is a rapidly developing field with several implementations now reaching the integrated circuit state

• Hybrid proposals should allow the transition from stationary to flying qubits for scalable architectures

• There are over 50 different quantum algorithms with more being developed all the time

• In time I am confident that quantum computing will become a practical reality